

**Study material on Sound – Part I**  
**SEM – IV Paper – 1D**  
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**Simple Harmonic Motion (or SHM)**

- SHM is a particular type of motion very common in nature.
- In SHM force acting on the particle is always directed towards a fixed point known as equilibrium position and the magnitude of force is directly proportional to the displacement of particle from the equilibrium position and is given by
- $F = -kx$
- where  $k$  is the force constant and negative sign shows that force opposes increase in  $x$ .
- This force is known as restoring force which takes the particle back towards the equilibrium position and opposes increase in displacement.
- S.I. unit of force constant  $k$  is N/m and magnitude of  $k$  depends on elastic properties of system under consideration.
- For understanding the nature of SHM consider a block of mass  $m$  whose one end is attached to a spring and another end is held stationary and this block is placed on a smooth horizontal surface shown below in the fig.a

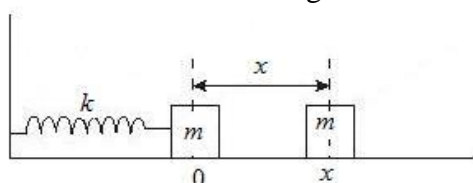


Figure a

- Motion of the body can be described with coordinate  $x$  taking  $x=0$  i.e. origin as the equilibrium position where the spring is neither stretched or compressed.
- We now take the block from its equilibrium position to a point  $P$  by stretching the spring by a distance  $OP=A$  and will then release it.
- After we release the block at point  $P$ , the restoring force acts on the block towards equilibrium position  $O$  and the block is then accelerated from point  $P$  towards point  $O$  as shown below in the fig.b

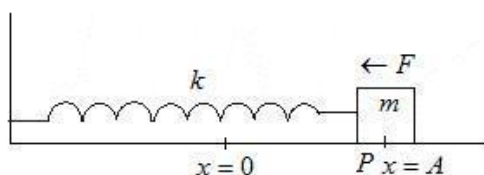


Figure b

- Now at equilibrium position this restoring force would become zero but the velocity of block increases as it reaches from point  $P$  to  $O$ .
- When the block reaches point  $O$  its velocity would be maximum and it then starts to move towards left of equilibrium position  $O$ .

- Now this time while going to the left of equilibrium position spring is compressed and the block moves to the point Q where it's velocity becomes zero.

### Equation of SHM

Consider any particle executing SHM with origin as it's equilibrium position under the influence of restoring force  $F = -kx$ , where  $k$  is the force constant and  $x$  is the displacement of particle from the equilibrium position.

Now since  $F = -kx$  is the restoring force and from Newton's law of motion force is give as  $F = ma$ , where  $m$  is the mass of the particle moving with acceleration  $a$ . Thus, acceleration of the particle is

$$\begin{aligned} \mathbf{a} &= \mathbf{F}/\mathbf{m} \\ &= -\mathbf{kx}/\mathbf{m} \end{aligned}$$

but we know that acceleration  $\mathbf{a} = \mathbf{dv}/\mathbf{dt} = \mathbf{d^2x}/\mathbf{dt^2}$

$$\Rightarrow \mathbf{d^2x}/\mathbf{dt^2} = -\mathbf{kx}/\mathbf{m} \quad (1)$$

This equation 1 is the equation of motion of SHM.

If we choose a constant  $\phi = \sqrt{(k/m)}$  then, equation 1 would become

$$\mathbf{d^2x}/\mathbf{dt^2} = -\phi^2\mathbf{x} \quad (2)$$

This equation is a differential equation which says that displacement  $x$  must be a function of time such that when it's second derivative is calculated the result must be negative constant multiplied by the original function.

Sine and cosine functions are the functions satisfying above requirement and are listed as follows  $x = A \sin\omega t$  (3a)

$$x = A \cos\omega t \quad (3b)$$

$$x = A \cos(\omega t + \phi) \quad (3c)$$

each one of equation 3a, 3b and 3c can be submitted on the left hand side of equation 2 and can then be solved for varification.

Conveniently, we choose equation 3c i.e., cosine form for representing displacement of particle at any time  $t$  from equilibrium position. Thus,

$$\mathbf{x} = \mathbf{A} \mathbf{cos}(\mathbf{\omega t} + \mathbf{\phi}) \quad (4)$$

and  $A$ ,  $\phi$  and  $\omega$  are all constants.

Fig below shows the displacement vs. time graph for phase  $\phi=0$ .

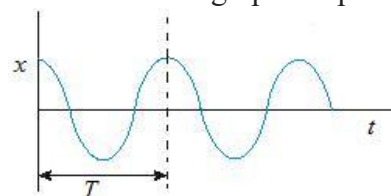


Figure 1

### Characterstics of SHM:

Here in this section we will learn about physical meaning of quantities like  $A$ ,  $T$ ,  $\omega$  and  $\phi$ .

#### (a) Amplitude

- Quantity  $A$  is known as amplitude of motion. it is a positive quantity and it's value depends on how oscillations were started.
- Amplitude is the magnitude of maximum value of displacement on either side from the equilibrium position.
- Since maximum and minimum values of any sine and cosine function are  $+1$  and  $-1$  , the maximum and minimum values of  $x$  in equation 4 are  $+A$  and  $-A$  respectively.
- Finally  $A$  is called the amplitude of SHM.

### b) Time period

Time interval during which the oscillation repeats itself is known as time period of oscillations and is denoted by  $T$ .

Since a particle in SHM repeats its motion in a regular interval  $T$  known as time period of oscillation so displacement  $x$  of particle should have same value at time  $t$  and  $t+T$ . Thus,

$$\cos(\omega t + \phi) = \cos(\omega(t+T) + \phi)$$

cosine function  $\cos(\omega t + \phi)$  will repeat its value if angle  $(\omega t + \phi)$  is increased by  $2\pi$  or any of its multiple. As  $T$  is the time period

$$(\omega(t+T) + \phi) = (\omega t + \phi) + 2\pi$$

$$\text{or, } T = 2\pi / \omega = 2\pi \sqrt{m/k} \quad (5)$$

Equation 5 gives the time period of oscillations.

Now the frequency of SHM is defined as the number of complete oscillations per unit time i.e., frequency is reciprocal of time period.

$$f = 1/T = 1/2\pi \sqrt{k/m} \quad (6)$$

$$\text{Thus, } \omega = 2\pi f = 2\pi/T \quad (7)$$

This quantity  $\omega$  is called the angular frequency of SHM. S.I. unit of  $T$  is s (seconds) and  $f$  is Hz (hertz)

### (c) Phase

- Quantity  $(\omega t + \phi)$  in equation (4) is known as phase of the motion and the constant  $\phi$  is known as initial phase i.e., phase at time  $t=0$ , or phase constant.
- Value of phase constant depends on displacement and velocity of particle at time  $t=0$ .
- The knowledge of phase constant enables us to know how far the particle is from equilibrium at time  $t=0$ . For example,

If  $\phi=0$  then from equation 4,  $x = A \cos \omega t$

that is displacement of oscillating particle is maximum, equal to  $A$  at  $t=0$  when the motion was started. Again, if  $\phi=\pi/2$  then from equation 4

$$x = A \cos(\omega t + \pi/2)$$

$$= A \sin \omega t$$

which means that displacement is zero at  $t=0$ .

- Variation of displacement of particle executing SHM is shown below in the fig

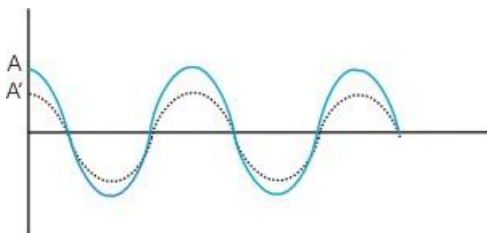


Figure 1a:- Figure shows the displacement Vs time graph for different amplitudes where  $A > A'$

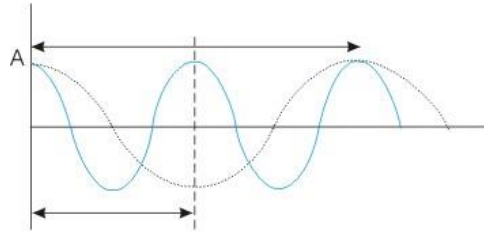


Figure 1b:- Figure shows the graph of SHM with different time periods where  $T' = T/2$

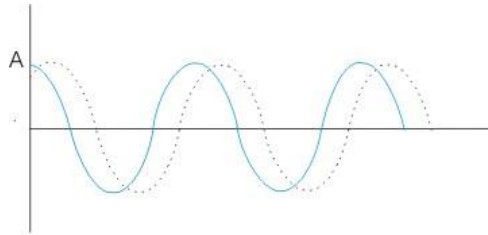


Figure 1c:- Figure shows SHM with different initial phase  $\phi$

#### (d) Velocity of SHM

- We know that velocity of a particle is given by  

$$\mathbf{v} = \mathbf{dx/dt}$$
- In SHM displacement of particle is given by  

$$\mathbf{x} = \mathbf{A \cos(\omega t + \phi)}$$
 now differentiating it with respect to t  

$$\mathbf{v} = \mathbf{dx/dt} = \mathbf{A\omega(-\sin(\omega t + \phi))} \quad (8)$$
- Here in equation 8 quantity  $A\omega$  is known as velocity amplitude and velocity of oscillating particle varies between the limits  $\pm\omega$ .
- From trigonometry we know that  

$$\mathbf{\cos^2\theta + \sin^2\theta = 1}$$

$$\Rightarrow \mathbf{A^2 \sin^2(\omega t + \phi) = A^2 - A^2 \cos^2(\omega t + \phi)}$$
 Or,  $\mathbf{\sin(\omega t + \phi) = [1 - x^2/A^2]} \quad (9)$ 
 putting this in equation 8 we get,  

$$\mathbf{v = -\omega A \left( 1 - \frac{x^2}{A^2} \right)^{1/2}}$$
- From the above equation we notice that when the displacement is maximum i.e.  $\pm A$  the velocity  $v = 0$ , because now the oscillator has to return to change its direction.
- Figure below shows the variation of velocity with time in SHM with initial phase  $\phi=0$ .

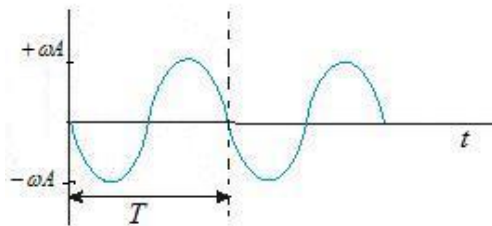


Figure 3

#### (e) Acceleration of SHM

- Again we know that acceleration of a particle is given by  

$$\mathbf{a} = \mathbf{dv/dt}$$
 where v is the velocity of particle executing motion.
- In SHM velocity of particle is given by,  

$$\mathbf{v} = \mathbf{-\omega \sin(\omega t + \phi)}$$
 differentiating this we get,

$$a = \frac{d}{dt}(-\omega A \sin(\omega t + \phi))$$

$$a = \frac{d}{dt}(-\omega A \sin(\omega t + \phi))$$

$$\text{or, } \mathbf{a = -\omega^2 A \cos(\omega t + \phi)} \quad (11)$$

- Equation (11) gives acceleration of particle executing simple harmonic motion and quantity  $\omega^2$  is called acceleration amplitude and the acceleration of oscillating particle varies between the limits  $\pm\omega^2 A$ .
- Putting equation (4) in (11) we get,  $\mathbf{a = -\omega^2 x}$  .....(12) which shows that acceleration is proportional to the displacement but in opposite direction.
- Thus from above equation we can see that when  $x$  is maximum ( $+A$  or  $-A$ ), the acceleration is also maximum ( $-\omega^2 A$  or  $+\omega^2 A$ ) but, is directed in direction opposite to that of displacement.
- Figure below shows the variation of acceleration of particle in SHM with time having initial phase  $\phi=0$ .

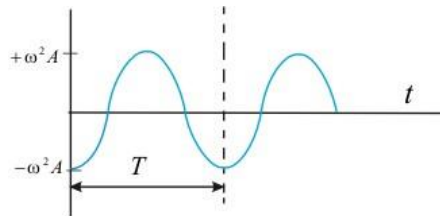


Figure :- 4

#### (f) Total energy in SHM

- When a system at rest is displaced from its equilibrium position by doing work on it, it gains potential energy and when it is released, it begins to move with a velocity and acquires kinetic energy.
- If  $m$  is the mass of system executing SHM then kinetic energy of system at any instant of time is

$$\mathbf{K = (1/2)mv^2} \quad \text{..... (13)}$$

putting equation 8 in 13 we get,

$$\begin{aligned} KE &= \frac{1}{2} m (-\omega A \sin(\omega t + \phi))^2 \\ &= \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi) \end{aligned} \quad (14)$$

- From equation (14) we see that Kinetic Energy of system varies periodically i.e., it is maximum ( $= \frac{1}{2} m \omega^2 A^2$ ) at the maximum value of velocity ( $\pm\omega A$ ) and at this time displacement is zero.
- When displacement is maximum ( $\pm A$ ), velocity of SHM is zero and hence kinetic energy is also zero and at these extreme points where kinetic energy  $K=0$ , all the energy is potential.
- At intermediate positions of lying between 0 and  $\pm A$ , the energy is partly kinetic and partly potential.

- To calculate potential energy at instant of time consider that  $x$  is the displacement of the system from its equilibrium at any time  $t$ .
- We know that potential energy of a system is given by the amount of work required to move system from position 0 to  $x$  under the action of applied force.
- Here force applied on the system must be just enough to oppose the restoring force  $-kx$  i.e., it should be equal to  $kx$ .
- Now work required to give infinitesimal displacement is  $dx=kx \cdot dx$ . Thus, total work required to displace the system from 0 to  $x$  is

$$= \int_0^x kx dx = \frac{1}{2} kx^2$$

thus,

$$PE = \frac{1}{2} kx^2$$

$$= \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \phi) \quad (15)$$

- where, from equation 5  $\omega = \sqrt{k/m}$  and displacement  $x = A \cos(\omega t + \phi)$ . From equation 14 and 15 we can calculate total energy of SHM which is given by,

$$E = KE + PE$$

$$= \frac{1}{2} m\omega^2 A^2 [\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

or,

$$E = \frac{1}{2} m\omega^2 A^2$$

- Thus, total energy of the oscillator remains constant as displacement is regained after every half cycle.
- If no energy is dissipated then all the potential energy becomes kinetic and vice versa.
- Figure below shows the variation of kinetic energy and potential energy of harmonic oscillator with time where phase  $\phi$  is set to zero for simplicity.

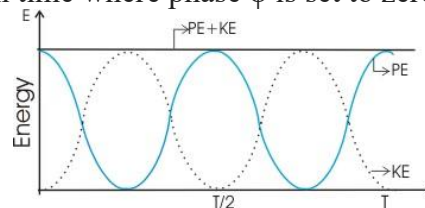


Figure 5:- Energy exchange in SHM

### Driven or Forced Harmonic oscillator:

- If an extra periodic force is applied on a damped harmonic oscillator, then the oscillating system is called driven or forced harmonic oscillator, and its oscillations are called forced oscillations.
- Such external periodic force can be represented by
 
$$F(t) = F_0 \cos \omega_f t \quad (16)$$
 where,  $F_0$  is the amplitude of the periodic force and  $\omega_f$  is the frequency of external force causing oscillations.
- Differential equation of motion under forced oscillations is

$$m \frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + kx = F_0 \cos \omega_f t \quad \dots\dots\dots(17)$$

- In this case particle will neither oscillate with its free undamped frequency nor with damped angular frequency rather it would be forced to oscillate with angular frequency  $\omega_f$  of applied force.
- When damped oscillator is set in forced motion, the initial motion is combination of damped oscillation and forced oscillations.
- After certain amount of time the amplitude of damped oscillations die out or become so small that they can be ignored and only forced oscillation remain and the motion is thus said to reached steady state.
- Solution of equation (17) is

$$\mathbf{x} = \mathbf{A} \cos(\omega_f t + \phi) \quad \dots\dots(18)$$

where A is the amplitude of oscillation of forced oscillator and  $\phi$  is the initial phase.

- In case of forced oscillations both amplitude A and initial phase  $\phi$  are fixed quantities depending on frequency  $\omega_f$  of applied force.
- Calculations show that amplitude

$$A = \frac{F_0}{\{m^2(\omega^2 - \omega_f^2) + \omega_f^2 \gamma\}^{1/2}} \quad \dots\dots(19)$$

and initial phase

$$\tan \phi = -v_0 / (\omega_f x_0)$$

where,  $x_0$  is displacement of particle at time  $t=0$ , the moment driven force is applied and  $v_0$  is the velocity of the particle at time  $t=0$ .

- When  $\omega_f$  is very close to  $\omega$ , then  $m(\omega^2 - \omega_f^2)^2$  would be much less than  $\omega_f \gamma$ , for any reasonable value of  $\gamma$ , then equation (19) becomes

$$\mathbf{A} = \mathbf{F}_0 / \gamma \omega_f \quad \dots\dots\dots (20)$$

- Thus, the maximum possible amplitude for a given driven frequency is governed by the driving frequency.
- This phenomenon of increase in amplitude when the driving force is close to natural frequency of oscillator is called RESONANCE.
- Thus, resonance occurs when frequency of applied force becomes equal to natural frequency of the oscillator without damping.

### Fourier Theorem:

**Fourier theorem** states that a periodic function  $f(x)$  which is reasonably continuous or has a finite number of finite discontinuities, may be expressed as the sum of a series of sine or cosine terms (called the **Fourier** series), each of which has specific amplitude and phase coefficients known as **Fourier** coefficients.

Analytically, the theorem may be written as

$$x(t) = a_0 + a_1 \cos \omega_0 t + b_1 \sin \omega_0 t + a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t + \dots$$

$$\text{or } x(t) = a_0 + \sum_{n=1}^{n=\infty} [ a_n \cos n\omega_0 t + b_n \sin n\omega_0 t ]$$

where,  $\omega_0 = \frac{2\pi}{T}$  is known as the fundamental frequency.

$n = 1, 2, 3, \dots$  an integer

$a_0$ ,  $a_n$  and  $b_n$  are calculated as :-

$$a_0 = \frac{1}{T} \int_0^T x(t) dt$$

$$a_n = \frac{2}{T} \int_0^T [x(t) \cos nw_0 t] dt$$

$$b_n = \frac{2}{T} \int_0^T [x(t) \sin nw_0 t] dt$$

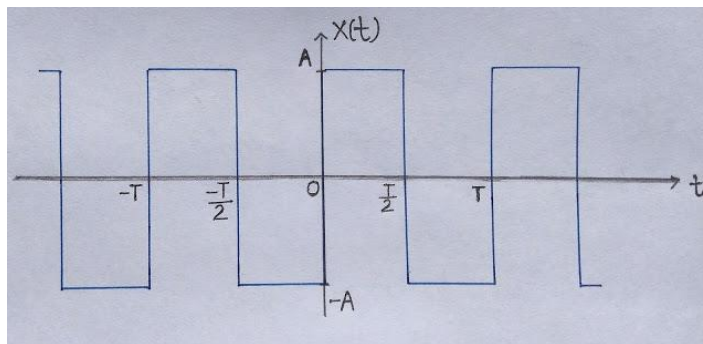
### 1. Fourier series for square wave:

The form of square wave is defined by

$$x(t) = A \text{ for } 0 < t < T/2$$

$$= -A \text{ for } T/2 < t < T$$

In this case,  $f(t)$  has a finite discontinuity at  $t = 0, T/2$  and  $T$ .



Fourier series representation of  $x(t)$  will be

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos nw_0 t + b_n \sin nw_0 t]$$

where,

$$\begin{aligned} a_0 &= \frac{1}{T} \int_0^T x(t) dt \\ &= \frac{1}{T} \left\{ \int_0^{T/2} A dt - \int_{T/2}^T A dt \right\} \\ &= 0 \end{aligned}$$

$$\begin{aligned} a_n &= \frac{2}{T} \int_0^T [x(t) \cos nw_0 t] dt \\ &= \frac{2}{T} \left\{ \int_0^{T/2} [A \cos nw_0 t] dt - \int_{T/2}^T [A \cos nw_0 t] dt \right\} \\ &= \frac{2}{T} \left\{ A \left[ \frac{\sin nw_0 t}{nw_0} \right]_0^{T/2} - A \left[ \frac{\sin nw_0 t}{nw_0} \right]_{T/2}^T \right\} \\ &= 0 \end{aligned}$$



$$\begin{aligned}
b_n &= \frac{2}{T} \int_0^T [x(t) \sin n\omega_0 t] dt \\
&= \frac{2}{T} \left\{ \int_0^{\frac{T}{2}} [A \sin n\omega_0 t] dt - \int_{\frac{T}{2}}^T [A \sin n\omega_0 t] dt \right\} \\
&= \frac{2}{T} \left\{ A \left[ \frac{-\cos n\omega_0 t}{n\omega_0} \right]_0^{\frac{T}{2}} - A \left[ \frac{-\cos n\omega_0 t}{n\omega_0} \right]_{\frac{T}{2}}^T \right\} \\
&= \frac{2A}{n\pi} \{1 - \cos n\pi\} \\
&= \frac{4A}{n\pi} \quad , \text{ where } n = \text{ odd}
\end{aligned}$$

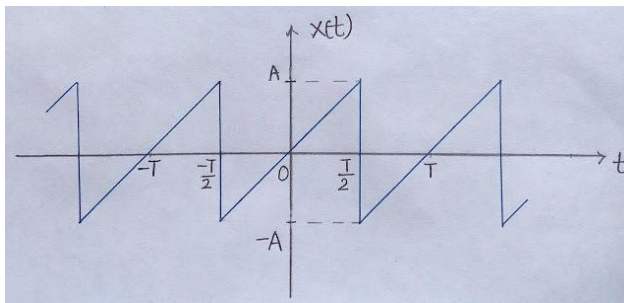
Therefore, *fourier series expression for square wave is*

$$\begin{aligned}
x(t) &= \sum_{n=1}^{n=\infty} b_n \sin n\omega_0 t \\
&= \frac{4A}{n\pi} \sum_{n=1}^{n=\infty} \sin n\omega_0 t \\
&= \frac{4A}{\pi} \sin \omega_0 t + \frac{4A}{3\pi} \sin 3\omega_0 t + \frac{4A}{5\pi} \sin 5\omega_0 t + \dots
\end{aligned}$$

## 2. Fourier series for sawtooth (triangular) wave:

For the saw-tooth wave as shown in the fig, the function  $x(t)$  increases relatively slowly from 0 to  $A$  in time  $T/2$  and then sharply falls to  $-A$  and the whole is repeated again and again.

In this case,  $x(t)/t = 2A/T$  Or,  $x(t) = 2At/T$  from  $t = -T/2$  to  $T/2$



Fourier series representation of  $x(t)$  will be

$$x(t) = a_0 + \sum_{n=1}^{n=\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

where,

$$\begin{aligned}
a_0 &= \frac{1}{T} \int_0^T x(t) dt \\
&= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{2At}{T} dt \\
&= 0
\end{aligned}$$

$$\begin{aligned}
a_n &= \frac{2}{T} \int_0^T [x(t) \cos nw_0 t] dt \\
&= \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{2At}{T} \cos nw_0 t dt \\
&= \frac{4A}{T^2} \left[ \frac{t \sin nw_0 t}{nw_0} + \frac{\cos nw_0 t}{n^2 w_0^2} \right]_{-\frac{T}{2}}^{\frac{T}{2}} \quad \dots \text{integration by parts} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
b_n &= \frac{2}{T} \int_0^T [x(t) \sin nw_0 t] dt \\
&= \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{2At}{T} \sin nw_0 t dt \\
&= \frac{4A}{T^2} \left[ \frac{-t \cos nw_0 t}{nw_0} + \frac{\sin nw_0 t}{n^2 w_0^2} \right]_{-\frac{T}{2}}^{\frac{T}{2}} \quad \dots \text{integration by parts} \\
&= \frac{-2A}{n\pi} \cos n\pi
\end{aligned}$$

Therefore, *fourier series expression for sawtooth (triangular) wave is*

$$\begin{aligned}
x(t) &= \sum_{n=1}^{n=\infty} b_n \sin nw_0 t \\
&= \frac{-2A}{n\pi} \sum_{n=1}^{n=\infty} \cos n\pi \sin nw_0 t \\
&= \frac{2A}{\pi} \sin w_0 t - \frac{2A}{2\pi} \sin 2w_0 t + \frac{2A}{3\pi} \sin 3w_0 t + \dots
\end{aligned}$$

### **Intensity of Sound:**

- Intensity of a sound wave is defined as the amount of sound energy passing through a unit area per second.
- Intensity of sound is given as:  $I = P/A$ , Where, I is the sound intensity, P is the acoustic power. A is the normal area to the direction of propagation.
- The amplitude of a sound decides its intensity, which in turn is perceived by the ear as loudness.

### **Loudness of sound:**

- Loudness is a measure of the response of the ear to the sound.
- The loudness of a sound is defined by its amplitude.
- If the amplitude of the sound wave is large, then the sound is said to be loud.
- It is directly proportional to the square of the amplitude of vibration. If the amplitude of the sound wave becomes double, then the loudness of the sound will be quadrupled.
- It is expressed in decibel (dB).
- Sounds above 80 dB becomes noise to human ears.

**Sound intensity levels** are quoted in decibels (dB) much more often than sound intensities in watts per meter squared. Decibels are the unit of choice in the scientific literature as well as in the popular media. The reasons for this choice of units are related to how we perceive sounds. How our ears perceive sound can be more accurately described by the logarithm of the intensity rather than directly to the intensity. The sound intensity level  $\beta$  in decibels of a sound having an intensity  $I$  in watts per meter squared is defined to be

$$\beta(\text{dB}) = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where  $I_0 = 10^{-12} \text{ W/m}^2$  is a reference intensity. In particular,  $I_0$  is the lowest or threshold intensity of sound a person with normal hearing can perceive at a frequency of 1000 Hz. Sound intensity level is not the same as intensity. Because  $\beta$  is defined in terms of a ratio, it is a unitless quantity telling you the level of the sound relative to a fixed standard ( $10^{-12} \text{ W/m}^2$  in this case). The units of decibels (dB) are used to indicate this ratio is multiplied by 10 in its definition. The bel, upon which the decibel is based, is named for Alexander Graham Bell, the inventor of the telephone.

The decibel level of a sound having the threshold intensity of  $10^{-12} \text{ W/m}^2$  is  $\beta = 0$  because  $\log_{10}1=0$ . That is, the threshold of hearing is 0 decibels.

Therefore, following is the table which explains the difference between loudness and intensity:

Loudness	Intensity
Loudness is the measure of response of the ear to the sound.	Intensity is the sound power per unit area.
Loudness is measured in decibels.	Intensity is measured in Watt per meter square.
Loudness is dependent on the sensitivity of the human ears.	Intensity is independent of the sensitivity of the human ears.
Loudness is a subjective quantity.	Intensity is an objective quantity.

### **Pitch of Sound**

- This depends on the frequency of vibration of the waves.
- If the frequency of vibration is higher, we say that the sound is shrill and has a high pitch. On the other hand, if the sound is said to have a lower pitch then it has a lower frequency of vibration.
- A bird produces high pitched sound whereas roaring of a lion is a low pitched sound.
- Voice of a woman has a high pitch than that of a man.

### **How do we make musical sounds?**

To make a sound, we need something that vibrates. If we want to make musical notes, you usually need the vibration to have an almost constant frequency, that means stable pitch. Many musical instruments make use of the vibrations of strings to produce the notes.

The physics of the stringed musical instruments is very simple. The notes played depend upon the string which is disturbed. The string can vary in length, its tension and its linear density (mass / length).

- Length L: different strings may have different lengths or the length of a string can be changed by using the fingers.
- Linear density  $\mu$  : different strings have differing linear density.
- String tension  $F_T$  : a string can be tuned by altering the string tension by using the tuning knobs.

It is only these three factors and how the string is disturbed that determines the vibrations of the string and the notes that it plays.

A string is disturbed and this sets up transverse waves travelling backward and forwards along the string due to reflections at the terminations of the string. The terminations act as nodes where the displacement of the string is always zero.

Only for a set of discrete frequencies, (natural or resonance frequencies of the string) can large amplitude standing waves be formed on the string to produce the required notes. The frequency of vibration of the string depends upon the wavelength of the wave and its speed of propagation.

$$f = v/\lambda \quad \dots\dots(1)$$

The speed of propagation of the waves along the string depends upon the string tension and the string's linear density whereas the wavelength is determined by the length of the string and the mode of vibration of the standing wave setup on the string.

$$v = \sqrt{\frac{F_T}{\mu}} \quad \dots\dots(2)$$

For the waves on strings, the boundary conditions are always fixed ends, therefore, upon reflection the wave is always inverted. The ends of the string correspond to nodes. The initial disturbance of the string sets up waves that travel along the string and are reflected. The resultant waveform is determined by the superposition of the multiple waves travelling backward and forward along the string. The resulting oscillation can form standing waves. The positions where the oscillations reach their maximum values are known as antinodes. At points where the amplitude of the oscillation is zero are called nodes – these points do not oscillate. For a standing wave, the distance between adjacent nodes or adjacent antinodes is  $\lambda/2$ .

When transverse oscillations are produced in a stretched string fasten at both ends, standing waves are setup. There must be nodes at the positions where the string is fastened. Hence, only oscillations are produced with appreciable amplitude when an integral number of half-wavelengths fit into the length of the string. This gives the condition

$$L = (n\lambda/2) \quad \dots\dots(3)$$

Therefore, from equation (1), the frequencies of the standing waves that can vibrate with appreciable amplitudes are

$$f = v/\lambda$$

or,  $f_n = n(v/2L)$  where  $n = 1,2,3,\dots \quad \dots\dots(4)$

The velocity  $v$  of the waves on the string depend only on the tension of the string  $F_T$  and linear density i.e., equation (2).

The frequencies are called the natural frequency of a string.

Therefore,  $f_n = \frac{n}{2L} \sqrt{\frac{F_T}{\mu}}$   $n = 1, 2, 3$

The fundamental frequency  $f_1$  ( $n = 1$ ) is the lowest frequency of oscillation.

$$f_1 = (v/2L) = \frac{1}{2L} \sqrt{\frac{F_T}{\mu}} \quad \dots\dots(5)$$

The natural frequencies are integral multiples of the fundamental frequency

$$f_n = n f_1$$

The natural frequencies are also called the harmonics:

- 1<sup>st</sup> harmonic  $n = 1$  fundamental
- 2<sup>nd</sup> harmonic  $n = 2$
- n<sup>th</sup> harmonic  $f_n$

The number  $n$  is called the mode number, and the value of  $n$  gives the natural or normal mode of the oscillation.

A musical tone is a steady periodic sound. A musical tone is characterized by its duration, pitch (frequency), intensity (loudness), and timbre or quality. Timbre or quality describes all the aspects of a musical sound other than pitch or loudness. Timbre is a subjective quantity and related to richness or perfection – music maybe heavy, light, murky, thin, smooth, clear, etc. For example, a note played by a violin has a brighter sound than the deeper sound from a viola when playing the same note. A simple tone or pure tone, has a sinusoidal waveform. A complex tone is a combination of two or more pure tones that have a periodic pattern of repetition.

When the string of a musical instrument is struck, bowed or hammered, many of the harmonics are excited simultaneously. The resulting sound is a superposition of the many tones differing in frequency. The fundamental (lowest frequency) determines the pitch of the sound. Therefore, we have no difficulty in distinguishing the tone of a violin and the tone from a viola of the same pitch – a different combinations of harmonic frequencies are excited when the violin and the viola the play same note.

### **Reverberation:**

Reverberation is the phenomenon of persistence of sound after it has been stopped as a result of multiple reflections from surfaces such as furniture, people, air etc. within a closed surface. These reflections build up with each reflection and decay gradually as they are absorbed by the surfaces of objects in the space enclosed.

It is same as the echo, but the distance between source of sound and also the obstacle through which it gets reflected is more less in case of this reverberation. The quantitative characterization of the reverberation is mainly done by using of the parameter called as reverberation time. Reverberation time is usually defined as length of the time where the sound decays by about 60 decibels starting from the initial level. In the process of reverberation, the time delay is said to be not less than 0.1 second i.e. the reflected form of wave reaches to the observer in more or less than 0.1 second. Hence this delay in perception of the sound and also the original sound is said to be very less and whereas the original sound will be still in the memory when this reflected sound is heard.

### **Advantages of Reverberation:**

Reverberations do wonders when it comes to musical symphonies and orchestra halls, when the right amount of reverberation is present, the sound quality gets enhanced drastically. This is the reason why sound engineers are appointed during the construction of these halls.

### **Disadvantages of Reverberation:**

If a room has about nearly no any sound absorbing surfaces like wall, roof and the floor, the sound is said to bounce back between the surfaces and also it takes a very long time as the sound dies. In such a room, the listener will then have a problem for registering the speaker. This is because he tends to hear both the direct sound as well as the repeated reflected sound waves. And also if these reverberations will be more excessive, the sound is said to run together with a mere loss of articulation, and it becomes muddy and also garbled.

### **How can we reduce reverberations?**

From our observation, we can say that, if the surface of the objects in the nearby enclosed space is covered with sound absorbing material, the reflected sound will decay much quicker and the listener will thus receive only the original sound. Porous materials such as mineral wool and fiberglass are examples of such absorbents. As the sound waves penetrate mineral wool, sound energy gets converted to heat through friction.

### **How to Calculate Reverberation Time?**

The first step to calculate the reverberation time is to calculate the Sabins with the below equation.

Formula for Sabins:

$$a = \sum S \alpha$$

Where:

$\Sigma$  = sabins (total room absorption at given frequency)

S = surface area of material (feet squared)

$\alpha$  = sound absorption coefficient at given frequency or the NRC

After we calculate 'a', we can then use the Sabine Formula to calculate the reverberation time.

Sabine Formula:

$$RT_{60} = 0.049 V/a$$

Where:

RT<sub>60</sub> = Reverberation Time

V = volume of the space (feet cubed)

a = sabins (total room absorption at given frequency)

This equation is based on the volume of the space and the total amount of absorption within a space. The total amount of absorption within a space is referred to as sabins. It is important to note that the absorption and surface area must be considered for every material within a space to calculate sabins.

### **Acoustic requirements of a good auditorium:-**

- The sound waves from the source should be of an adequate intensity.

- The sound should spread everywhere inside the hall evenly and should be audible enough everywhere.
- The sound notes should be clear and distortions should be minimum.
- Undesired noise interfere with the sound from the source should be minimized.
- Sound distortion due to shape and size of an object present in the auditorium must be reduced.
- Proper reverberation time according to preferred values is also important.

**Measures to be taken:-**

- The site for the auditorium must be selected carefully and noisy areas like near the railway tracks or airports must be avoided.
- Size of the auditorium must be optimum as small size would create irregular distribution of sound while large size will result into weaker intensity sound waves and larger reverberation time.
- Avoiding echoes and focusing of sound waves is important as they cause non uniform sound level within the room which is undesirable.
- In order to achieve this, we can make use of scatters, convex (non-focusing) surfaces on balcony fronts, sound absorption materials on concave surfaces and flat surfaces, carpets, and upholstered seats.
- The reflections from the rear walls and ceiling may cause disturbance, hence it must be reduced by using absorbents.
- Reverberation time can be controlled by proper use of absorbing materials, optimum number of audience, presence of optimum number of windows, etc.
- Railings or staircase or any regularly spaced objects will reflect the sound waves from the source and produce echoes in regular successions. This phenomenon is called Echelon effect. This can be reduced by covering such surfaces by a thick carpets or absorbents.